Name: ____________________________________ Date: __________________

A2CC: Sequences

Sequences are extremely important in mathematics, both theoretical and applied. A **sequence** is formally defined as **a function that has as its domain the set the set of positive integers**, i.e. $\{1, 2, 3, ..., n\}$.

***Exercise* #1:** A sequence is defined by the equation $a(n) = 2n - 1$.

(a) Find the first three terms of this sequence, denoted by $a_1$, $a_2$, and $a_3$.

(b) Which term has a value of 53?

(c) Explain why there will not be a term that has a value of 70.

Recall that sequences can also be described by using **recursive definitions**. When a sequence is defined recursively, terms are found by operations on previous terms.

***Exercise* #2:** A sequence is defined by the recursive formula: $f(n) = f(n-1) + 5$ with $f(1) = -2$.

(a) Generate the first five terms of this sequence. Label each term with proper **function** notation.

(b) Determine the value of $\boldsymbol{f(20)}$. Hint – think about how many times you have added 5 to $\boldsymbol{-2}$.

***Exercise* #3:** Determine a recursive definition, in terms of $f(n)$, for the sequence shown below. Be sure to include a starting value.

5, 10, 20, 40, 80, 160, …

***Exercise* #4:** For the recursively defined sequence $t_n = \left(t_{n-1}\right)^2 + 2$ and $t_1 = 2$, the value of $t_4$ is

(1) 18 (3) 456

(2) 38 (4) 1446

***Exercise* #5:** One of the most well-known sequences is the Fibonacci, which is defined recursively using two previous terms. Its definition is given below.

$$f(n) = f(n-1) + f(n-2) \text{ and } f(1) = 1 \text{ and } f(2) = 1$$

Generate values for $f(3)$, $f(4)$, $f(5)$, and $f(6)$ (in other words, then next four terms of this sequence).

It is often possible to find algebraic formulas for simple sequences, and this skill should be practiced.

***Exercise* #6:** Find an algebraic formula $a(n)$, similar to that in *Exercise* #1, for each of the following sequences. Recall that the domain that you map from will be the set $\{1, 2, 3, ..., n\}$.

(a) $4, 5, 6, 7, ...$ (b) $2, 4, 8, 16, ...$ (c) $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, ...$

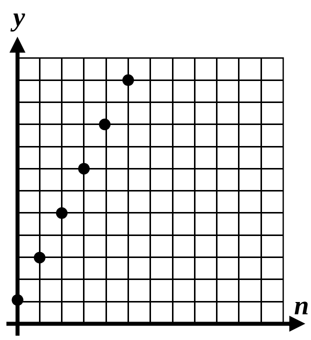
(d) $-1, 1, -1, 1, \ldots$

(e) $10, 15, 20, 25, \ldots$

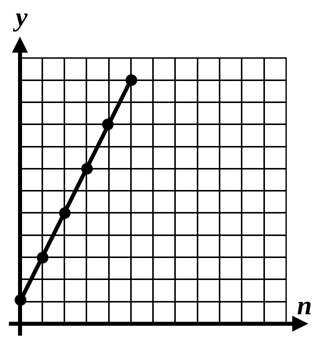
(f) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

***Exercise* #7:** Which of the following would represent the graph of the sequence $a_n = 2n+1$? Explain your choice.

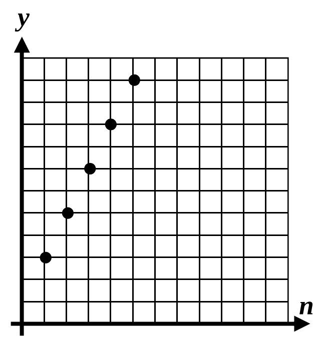
(1)



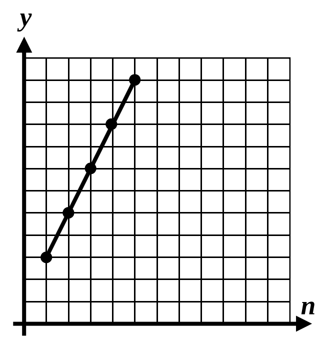

(2)




(3)




(4)




Explanation:

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A2CC: Sequences

# HOMEWORK

1. Given each of the following sequences defined by formulas, determine and label the first four terms. A variety of different notations is used below for practice purposes.

   (a) $f(n)=7n+2$ (b) $a_n=n^2-5$ (c) $t(n)=\left(\frac{2}{3}\right)^n$ (d) $t_n=\dfrac{1}{n+1}$

2. Sequences below are defined recursively. Determine and label the **next** three terms of the sequence.

   (a) $f(1)=4$ and $f(n)=f(n-1)+8$ (b) $a(n)=a(n-1)\cdot\frac{1}{2}$ and $a(1)=24$

   (c) $b_n=b_{n-1}+2n$ with $b_1=5$ (d) $f(n)=2f(n-1)-n^2$ and $f(1)=4$

3. Given the sequence 7, 11, 15, 19, ..., which of the following represents a formula that will generate it?

   (1) $a(n)=4n+7$ (3) $a(n)=3n+7$

   (2) $a(n)=3n+4$ (4) $a(n)=4n+3$

4. A recursive sequence is defined by $a_{n+1}=2a_n-a_{n-1}$ with $a_1=0$ and $a_2=1$. Which of the following represents the value of $a_5$?

   (1) 8 (3) 3

   (2) $-7$ (4) 4

5. Which of the following formulas would represent the sequence 10, 20, 40, 80, 160, …

   (1) $a_n=10^n$ (3) $a_n=5(2)^n$

   (2) $a_n=10(2)^n$ (4) $a_n=2n+10$

6. For each of the following sequences, determine an algebraic formula, similar to *Exercise* #4, that defines the sequence.

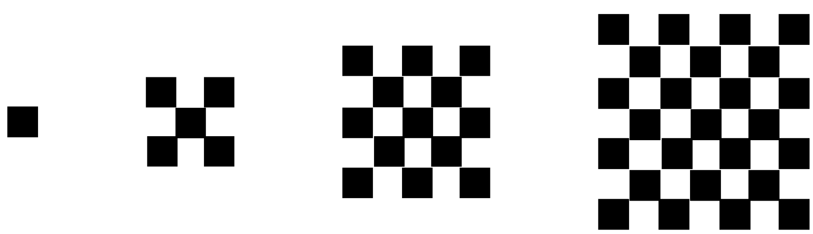
   (a) 5, 10, 15, 20, …  (b) 3, 9, 27, 81, …  (c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

7. For each of the following sequences, state a recursive definition. Be sure to include a starting value or values.

   (a) 8, 6, 4, 2, …  (b) 2, 6, 18, 54, …  (c) $2, -2, 2, -2, \ldots$

## APPLICATIONS

8. A tiling pattern is created from a single square and then expanded as shown. If the number of squares in each pattern defines a sequence, then determine the number of squares in the seventh pattern. Explain how you arrived at your choice. Can you write a recursive definition for the pattern?



## REASONING

9. Consider a sequence defined similarly to the Fibonacci, but with a slight twist:

$$f(n) = f(n-1) - f(n-2) \text{ with } f(1) = 2 \text{ and } f(2) = 5$$

   Generate terms $f(3)$, $f(4)$, $f(5)$, $f(6)$, $f(7)$, and $f(8)$. Then, determine the value of $f(25)$.